Chapter 4

Effects of Mathematics Language on Children's Mathematics Achievement and Central Conceptual Knowledge

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The primary goal of this chapter is to consider the relation among mathematics achievement, mathematics language, and central numerical knowledge. My colleagues and I have been examining how culturally bound factors such as characteristics of numerical language impact children's learning of school mathematics content (e.g., Miura & Okamoto, 2003). At the same time, from a neo-Piagetian perspective, my colleagues and I have examined the development of central numerical knowledge across cultures (e.g., Okamoto, Case, Bleiker, & Henderson, 1996). The former line of work suggests that mathematics achievement is impacted by cultural factors. The latter suggests that despite achievement differences, children develop numerical knowledge at similar rates. These two suggestions appear contradictory. How do I reconcile these seemingly contradictory suggestions?

My current view is that children growing up in cultures as different as Japan and the United States, for example, develop foundational numerical concepts at about the same rate. However, achievement gaps in mathematics reflect different problem-solving experiences children encounter in and out of school that are valued by particular cultures (see also Chapter 3 by LeFevre et al. and Chapter 5 by Opfer et al.). Differences in numerical language characteristics may provide qualitatively different problem-solving experiences. This is particularly so for young children who are just beginning formal schooling. For these children, differences in numerical language characteristics impact both their learning of school mathematics content and development of their central numerical knowledge.

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As such, this chapter is organized into two broad sections. The first focuses on mathematics achievement differences and factors that may explain these differences. Evidence is presented in favor of numerical/mathematical language as a potential factor to explain the achievement gap. In the second section, I describe a series of cross-cultural studies that examined the relation between the development of central conceptual knowledge and the attainment of specific skills in particular domains. With the exception of one anomaly, the data point to similar rates of development of central conceptual knowledge. The anomaly was found in Japanese 6-year olds' numerical knowledge. These children showed the development of numerical knowledge 1–2 years ahead of their US peers. This anomaly will be explained in terms of different characteristics of numerical language.

MATHEMATICS LANGUAGE AND ACHIEVEMENT

Achievement gaps between US and East Asian students have been documented since the publication of the results from the First International Mathematics Study (FIMS; Husén, 1967). Although US students have shown an upward trend in international standings over the last five cycles of the Trends in International Mathematics and Science Study (TIMSS), their performance remains substantially lower than their East Asian counterparts (Beaton et al., 1997; Mullis & Martin, 2008; Mullis et al., 1998, 2000; Mullis, Martin, Foy, & Arora, 2012; Mullis, Martin, Gonzalez, & Chrostowski, 2005). In fact, the most recent TIMSS results showed that at the fourth-grade level, US students were 10–14 percentage points behind their East Asian peers (Mullis et al., 2012). Similar results have been obtained in the Program for International Student Assessment. The most recent data from the 2012 study show that only 9% of US 15-year-old students reached the highest category of performance (i.e., scores 607 or above out of 1000 maximum) in mathematics literacy whereas 55% of Chinese students in Shanghai did so (retrieved from http://nces.ed.gov/surveys/pisa/pisa2012/index.asp).

In both these large-scale international comparisons, East Asian students show strengths in the attainment of mathematics content deemed important by the experts, including those who constructed the measures. These results have inspired many researchers to attempt to explain why such differences exist (see Chapter 5 by Opfer et al.).

Early research comparing characteristics of schools in Japan, Taiwan, and the United States documented that Chinese and Japanese students spent more time in school in general and, in particular, studying mathematics than did their US counterparts (e.g., Stigler, Lee, & Stevenson, 1987). Subsequent research began to focus on teaching practices. This line of work is most clearly represented in Stigler and Hiebert's (1999) book, *The Teaching Gap*. They argued that differences in teaching methods not teachers' ability to teach led to varying learning opportunities for US and Japanese students. For example, the video analysis of teaching practices in the United States, Germany, and Japan (TIMSS videotape study; Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999) showed a wide disparity in the types of mathematical activities in which students were engaged: during seatwork, Japanese students spent far more time inventing, analyzing, and proving than did their US and German peers, who, in turn, spent almost their entire time practicing routine procedures. Japanese practices documented in the TIMSS videotape study are consistent with the practices recommended by the National Council of Teachers of Mathematics (2000).

It is hardly surprising that variations in school experiences, in particular mathematics classroom experiences, account for achievement differences. However, marked differences in mathematics performance have been found as early as in kindergarten when the impact of teaching effectiveness and other school-related factors is minimal (e.g., Stevenson, Lee, & Stigler, 1986). This finding calls for consideration of factors other than teaching practices to account for early differences in mathematics achievement (see Chapter 5 by Opfer et al.).

Mathematics Language

The factor of central interest in explaining achievement differences is variations in how mathematical terms are expressed in different language groups. Our focus has been to examine the impact of numerical language characteristics between East Asian (e.g., Japanese) and non-East Asian (e.g., English) languages (e.g., Miura & Okamoto, 2003). Although written numerical symbols (i.e., Arabic numerals) are practically universal across cultures, spoken words associated with written symbols differ from one language to another (e.g., 1 is spoken as "yi," "ichi," and "il" in Chinese, Japanese, and Korean, respectively). As in any language, different words are used to distinguish the base sequence of numbers from 1 to 10. The number names above 10, however, show interesting variations.

The number naming systems of East Asian languages such as Chinese, Japanese, and Korean have their roots in ancient Chinese. In these languages, the number names above 10 are congruent with the traditional base-10 numeration system (see Table 1). That is, a number word for any given two-digit number can be generated from a set of base-10 rules and a base sequence of number names. For example, 11 is spoken as "ju-ichi" (i.e., ten-one) in Japanese as opposed to "eleven" in English. The latter is a new word for children to memorize. Similarly, the teen words in English (e.g., fourteen) have the single-digit number word first, followed by the word that signals "ten." In Korean, 14 is "shib-sah," which means "ten-four." The numbers 20 and above can also be generated by applying the base-10 rules in these East Asian languages. In Chinese, 55 is spoken as "wu-shi-wu," which is "five-tens-five." In English, however, it is "fifty-five," which implies "fifty" as a *chunk*, not five tens. It should be clear from these examples that Chinese, Japanese, and Korean children need to memorize only the number names for 1–10,

TABLE 1 Number words in English, Japanese, Chinese, and Korean

Number	English	Japanese	Chinese	Korean
1	One	Ichi	Yi	Il
2	Two	Ni	Er	Ee
3	Three	San	San	Sam
4	Four	Shi	Si	Sah
5	Five	Go	Wu	Oh
6	Six	Roku	Liu	Yook
7	Seven	Shichi	Qi	Chil
8	Eight	Hachi	Ba	Pal
9	Nine	Kyuu	Jiu	Goo
10	Ten	Juu	Shi	Shib
11	Eleven	Juu ichi	Shi yi	Shib il
12	Twelve	Juu ni	Shi er	Shib ee
13	Thirteen	Juu san	Shi san	Shib sam
14	Fourteen	Juu shi	Shi si	Shib sah
15	Fifteen	Juu go	Shi wu	Shib oh
16	Sixteen	Juu roku	Shi liu	Shib yook
17	Seventeen	Juu shichi	Shi qi	Shib chil
18	Eighteen	Juu hachi	Shi ba	Shib pal
19	Nineteen	Juu kyuu	Shi jiu	Shib goo
20	Twenty	Ni juu	Er shi	Ee shib
22	Twenty two	Ni juu ni	Er shi er	Ee shib ee
33	Thirty three	San juu san	San shi san	Sam shib sam
44	Forty four	Shi juu shi	Si shi si	Sah shib sah
55	Fifty five	Go juu go	Wu shi wu	Oh shib oh
66	Sixty six	Roku juu roku	Liu shi lie	Yook shib yook
77	Seventy seven	Shichi juu shichi	Qi shi qi	Chil shib chil
88	Eighty eight	Hachi juu hachi	Ba shi ba	Pal shib pal
99	Ninety nine	Kyuu juu kyuu	Jiu shi jiu	Goo shib goo
100	One hundred	Hyaku	Bai	Baek

and the rest can be generated by using these names and applying the base-10 rules.[1]

On a side note, English is not the only language that does not follow the base-10 rules. Spanish is somewhat closer to English in that new number names are used to express numbers 11 through 15 and decade names do not suggest multiples of tens. There are number names in Spanish that are similar to those of East Asian languages: the numbers 16–19 are spoken as ten and single-digit number (e.g., 17 is "diecisiete," which is ten-seven). Other interesting number names include 43 in German (which means three and forty), 80 in French (four times twenty), and 50 in Danish ((two and a half) times twenty). (See Chapter 3 by LeFevre et al. for related information about Turkish as well.)

Cognitive Organization of Numbers

Given a variation in characteristics of numerical language, it is plausible to expect that such a variation may have differentiated effects on how children mentally organize numbers. In a series of studies, Miura and colleagues (e.g., Miura, 1987; Miura & Okamoto, 1989) pursued this very question. That is, they wondered if children speaking regular or irregular number words develop different mental representations of two-digit numbers.

The method used in these studies involved commercially available base-10 blocks (see Fig. 1). Children were shown small cubes (one-blocks) and

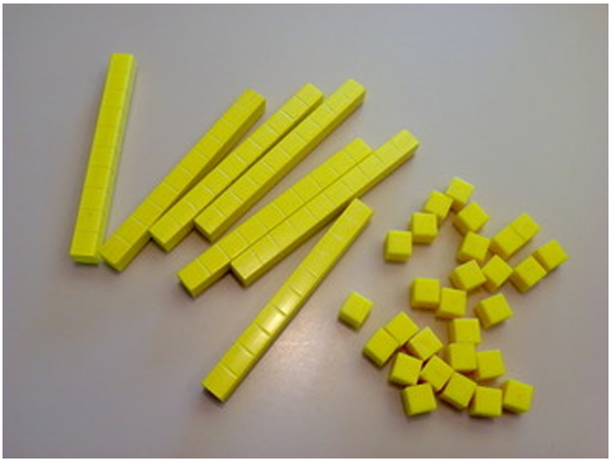


FIG. 1 One blocks (right) and ten blocks (left).

1. Although children's acquisition of two digit numbers is the focus of this chapter, the number names in these East Asian languages remain regular beyond 99. In Japanese, for example, the numbers up to 99,999 can be generated by applying the base 10 rules.

rectangular prisms (ten-blocks). They then saw a demonstration in which ten small cubes (10 one-blocks) were equivalent to one rectangular prism (1 ten-block). They were next asked to construct two-digit numbers using one- and ten-blocks. This was considered as children's *spontaneous* preference to represent the number. Once children complete the task, they were shown their own construction of each number and asked if they could think of another way to show the same number. In the initial study conducted by Miura (1987), Japanese-speaking first graders[2] spontaneously used the precise combinations of ten- and one-blocks to show two-digit numbers (e.g., 1 ten-block and 3 one-blocks for 13). In contrast, English-speaking first graders rarely chose to use ten-blocks to represent two-digit numbers. Instead, they only used one-blocks (e.g., 13 one-blocks for 13).

Miura's (1987) initial study sparked interest among those who study the relation between language and thought in general and mathematics in particular. Replications, however, were necessary to determine if Miura's initial findings would hold up. In later replication studies, Miura and colleagues recruited first graders, who had not been taught two-digit numbers, from a wider range of nations. From East Asia, they recruited children from China, Japan, and Korea. From non-East Asia, children from France and Sweden, in addition to the United States, were included. The Swedish number naming system is almost identical to the English system. The French system shares some characteristics similar to English but has its own unique number names as mentioned earlier. The results of these studies showed remarkable similarity to Miura's initial findings. For example, when asked to construct 28, the overwhelming majority of Chinese-, Japanese-, and Korean-speaking first graders preferred to use 2 ten-blocks and 8 one-blocks (i.e., the base-10 construction), whereas almost all English-speaking counterparts selected 28 one-blocks as their first construction (Miura, Kim, Chang, & Okamoto, 1988). French- and Swedish-speaking first graders were found to be much like English-speaking participants in that they also selected one-blocks to show two-digit numbers during the first attempt (Miura, Okamoto, Kim, Steere, & Fayol, 1993).

It is possible that children's initial construction does not tell the whole story about their cognitive representation of number. Miura and colleagues (Miura et al., 1988) thus asked children to come up with another way to show the same number. Considering the two constructions children made, they found large differences in how English- and East Asian-speaking children conceived of two-digit numbers: 76%, 79%, and 98% of Chinese, Japanese, and Korean first graders, respectively, made base-10 constructions for all of the numbers in either the first or second attempt or both, whereas only 13% of English-speaking first graders did so. Furthermore, about half of the English-speaking children did not use the base-10 construction at all.

2. The participants resided in California, attending a Saturday Japanese school. All of them spoke Japanese fluently, and their home language was Japanese.

Somewhat similar results were found in the 1993 study: 67% and 100% of Japanese and Korean children, respectively, made base-10 constructions for all of the numbers in either the first or second attempt or both. In contrast, 35%, 65%, and 8% of French, Swedish, and US children did so (Miura et al., 1993). The Swedish result was close to that of Japanese. When the first attempt was considered, however, over 98% of the Swedish first graders chose to use one-blocks only, whereas over 72% of the Japanese counterparts made base-10 constructions.

Across the three studies, common findings are that East Asian speakers differed significantly from non-East Asian speakers in the kinds of constructions they made for two-digit numbers. From these results, Miura and colleagues inferred that ancient Chinese-based number systems influenced the way that young children mentally represent two-digit numbers.

A major critique of these studies is that the results could be due to cultural factors other than linguistic differences (cf., Towse, Muldoon, & Simms, 2015; Towse & Saxton, 1997). Because the participants came from different cultures, children's cultural and educational experiences could be confounding factors. Dowker, Bala, and Lloyd (2008) tested this possibility with children who lived in the same region of Wales. Children in Wales speak Welsh or English or both. The number system in Welsh is congruent with the base-10 system. Dowker et al. found that Welsh-speaking children understood two-digit numbers more accurately than their English-speaking counterparts. Thus, the results from their study provide support for the idea that there is a causal link between numerical language and children's numerical thinking.

Numerical Language and Estimation of Numerical Magnitudes

More recently, several cross-cultural studies were conducted to examine the link between numerical language and children's understanding of numerical magnitudes. Siegler and Mu (2008), for example, asked Chinese and US kindergartners to estimate single- and two-digit numbers on a 0–100 number line. Children were told the locations of 0 (far left) and 100 (far right) on the number line and asked to show where each of the target numbers would go. The literature on this topic has shown that kindergartners in the United States typically overestimate smaller numbers and underestimate larger numbers, resulting in an estimation pattern that is best described as logarithmic (e.g., Siegler & Booth, 2004). By second grade, children gain better understanding of the correspondence between numerals and their magnitudes. Thus, their estimation pattern is best described as linear. Siegler and Mu's study found that Chinese kindergartners already showed a linear pattern of numerical estimation. US kindergartners' estimation, however, was logarithmic as has been found in other studies (see Chapter 5 by Opfer et al. for a thorough analysis of number-line estimation and cross-cultural differences in this skill).

Although Siegler and Mu (2008) did not attribute their findings to regular and irregular counting systems of Chinese and English, respectively, other studies have made an explicit link to the linguistic factor. Dowker and Roberts (2015) examined this question with Welsh- and English-speaking 6-year-old children who lived in the same region of Wales. Similar to Siegler and Mu, they asked children to mark where a particular number would go on the 0–100 number line. They found that overall, Welsh-speaking children were more accurate in their estimation than their English-speaking counterparts. Their further analyses revealed interesting findings. For the numbers under 10, Welsh- and English-speaking children did not differ in their estimation. However, for numbers above 10 and in particular above 20, Welsh-speaking children were significantly more accurate than their English-speaking counterparts. Similar findings were reported between German- and Italian-speaking children (Helmreich et al., 2011). The German counting system includes inversion properties (e.g., 48 is spoken as "eight and forty"), whereas no such properties appear in Italian. Helmreich et al. found that Italian-speaking children were more accurate in number-line estimation than their German-speaking counterparts. Thus, the cross-cultural comparisons of numerical estimation further provide support for the link between numerical language and children's numerical thinking.

Relation of Numerical Language to Mathematics Achievement

The studies discussed above provided evidence that numerical language characteristics influence the way children understand two-digit numbers. The question then is in what ways, if any, differences in cognitive representation of number might be related to mathematics performance. This question was addressed in Miura and Okamoto's (1989) study, albeit indirectly. They assessed English- and Japanese-speaking children's mathematics achievement and their cognitive representation of number. The achievement measures were not the same across cultures. In the United States, it was the Educational Records Bureau's standardized achievement test. In Japan, it was based on teacher ratings. Because the two measures were drastically different, analyses were carried out only within each nation. Miura and Okamoto reported that being able to show two-digit numbers in two different ways (e.g., the correct combination of ten- and one-blocks vs one-blocks only) was significantly correlated with mathematics achievement ($r = 0.49$ and $r = 0.37$ for the Japanese and United States participants, respectively).

Place-Value Understanding

Place value refers to the idea that the value of a digit depends on its relative position in a number. That is, the value of a given digit in a multidigit numeral depends on the face value of the digit (0–9) and on its position in the numeral, with the value of each position increasing by powers of 10 from right to left. In other words, to say that a child has an understanding of place value requires that the child understands the rules of the base-10 numeration system (e.g., the "2" in

28 stands for 20). It follows then that children who think of numbers as consisting of tens and ones are likely to show strong place value understanding.

Miura and colleagues tested this possibility (Miura & Okamoto, 1989; Miura et al., 1993). In addition to constructing two-digit numbers using base-10 blocks, they asked a series of questions about children's understanding of place value. A relatively easy question involved showing children a card with a two-digit number written on it and asking them to point to the numeral in the one's and ten's position. A slightly more difficult question entailed presenting children with 3 ten-blocks and 12 one-blocks and asking them to write the number the blocks represented. A more difficult question was posed when children were shown 3 clear plastic cups and 13 one-blocks. They were then asked to put four one-blocks in each cup. This resulted in *3* cups (with four one-blocks in each cup) and *1* one-block remaining. Thus, children saw *1* cube and *3* cups. The experimenter then showed a card with 13 written on it and pointed to the "1" and the "3" in turn and asked what each meant in relation to the cups and cubes in front of them. Children with weak or no understanding of place value were often distracted by the visual display and said that the 1 in 13 referred to 1 cube and the 3 in 13 referred to 3 cups.

When Miura and Okamoto (1989) compared US and Japanese first graders' place value understanding, they found a statistically significant difference in favor of Japanese children. In fact, one-half of English-speaking first graders were unable to respond to any of the place value questions, whereas all of the Japanese counterparts responded to at least one problem correctly, and 42% of them solved all of the problems correctly. Due, in part, to the small sample, no statistically significant correlations between cognitive representation of number and place value understanding were found in either group. In another study in which first graders from a greater number of nations were recruited, Miura et al. (1993) reported that East Asian-speaking children from Japan and Korea performed significantly better on the place value task than those from France, Sweden, and the United States. To examine the relation between cognitive representation of number and place value understanding, all children were combined for a regression analysis. The results showed that 58% of the variance in place value performance was explained by children's initial base-10 constructions as opposed to their second construction.

Although the evidence is correlational, the results across the two studies suggest that children's spontaneous preference to represent two-digit numbers as consisting of tens and ones (as opposed to only ones) facilitates the acquisition of place value of two-digit numbers, which, in theory, should provide a strong basis for generalizing the rules of base-10 for two-digit numbers to multidigit numbers. This should, in turn, facilitate children's understanding of later arithmetic performance, including multidigit column addition and subtraction. In fact, Moeller, Pixner, Zuber, Kaufmann, and Nuerk (2011) found that early mastery of base-10 knowledge in first grade was a reliable predictor of later arithmetic performance in third grade. The opposite has also been found to be true, in that children who struggle to develop place value understanding early on struggle to do well on multidigit addition and subtraction computations with regrouping (e.g., Ho & Cheng, 1997). Taken together, it seems reasonable to conclude that the compatibility of the spoken number words with the base-10 rules influences the way young children organize two-digit numbers, which, in turn, has a significant impact on the rate at which children form an understanding of place value and multidigit number arithmetic (Alsawaie, 2004; Dowker et al., 2008; Ho & Fuson, 1998).

Numerical Language and Instructional Methods

Multidigit column addition and subtraction are difficult for children who have yet to understand place value. These children tend to treat digits in different columns as if they are independent. When asked what the "2" in 28 means, English-speaking children without place value understanding would say "two" not "twenty." When faced with column addition with regrouping (i.e., carrying and borrowing) such as 34 − 18, the lack of place value understanding results in answers such as 24 ("smaller-from-larger bug"; see Brown & Burton, 1978). Teachers do their best to teach children how to borrow and carry so that they will not make this sort of error. Miura and Okamoto (1999) pointed out that different teaching methods could result in part from characteristics of numerical language. US children are typically taught 34 − 18 in the following manner (see Table 2a):

> *You can't take the 8 from the 4. So you borrow 10 from the 30 to make 14 (fourteen). Now you can take the 8 from the 14 and you get 6.*

This method is also feasible in Japan. In addition to this method, however, Japanese children understand the method that takes advantage of its base-10 numerical language (see Table 2b):

> *You can't take the 8 from the 4. So you borrow 10 from the 30 to make 14 (ten four). 10 take away 8 is 2. You add the 4 to the 2 and you get 6.*

The Japanese method naturally arises from the base-10 language characteristics. Teachers would not have to explain why they break 14 into 10 and 4. Because 14 is "ten-four," it makes sense to think of 14 as consisting of 1

TABLE 2 Subtraction strategies

(a) Strategy based on *fourteen*		(b) Strategy based on *ten-four*	
2 10 34 −18 6	14 − 8 = 6	2 10 34 −18 6	10 − 8 = 2 2 + 4 = 6

ten and 4 ones. Young children have more practice with number combinations up to 10 and adding two single-digit numbers. Thus, the Japanese method is less prone to errors. The first method, however, requires that children know number facts beyond 10 which is more advanced or use other strategies such as "counting up from" (e.g., $14-8$ is accomplished by counting up from 8 to 14), which is more prone to counting errors. These examples suggest that differences in how number words are spoken have implications for how arithmetic computation is taught.

Fraction Terms

Many children and adults for that matter find fractions difficult (Ma, 1999; Moseley & Okamoto, 2008; Moseley, Okamoto, & Ishida, 2007; Siegler et al., 2010). It has been reported that only 13% of US fifth graders reached fraction proficiency (Princiotta, Flanagan, & Germino Hausken, 2006). This is of serious concern as understanding of fractions has been identified as a strong predictor of later success in mathematics in the United States and the United Kingdom (Siegler et al., 2012). When children are first learning fractions, they often apply what they know about whole number arithmetic to fraction arithmetic. This is known in the literature as the "whole number bias" (e.g., Ni & Zhou, 2005; Vosniadou, Vamvakoussi, & Skopeliti, 2008). For one thing, children do not see a fraction notation as referring to a particular quantity. Instead, they treat the numerator and denominator as two separate whole numbers. Thus, $2/3+1/3$ could result in an answer 3/6 (e.g., Mack, 1990). English fraction terms are not particularly helpful in overcoming this misconception. For example, 2/3 is spoken as "two-thirds," which does not readily convey any fraction meaning for children who are learning fractions for the first time. In contrast, in East Asian languages, 2/3 is spoken as "of (the whole) divided into three, (take) two." This conveys the notion of part-whole. Of course, part-whole is just one interpretation of rational numbers (Moss & Case, 1999), and full-fledged understanding of rational numbers requires expanded and multiple interpretations (Behr, Harel, Post, & Lesh, 1992; Kieren, 1993). Nonetheless, part-whole is one of the meanings that fractional notations convey, and children need to develop this understanding as well.

Miura, Okamoto, Vlahovic-Stetic, Kim, and Han (1999) hypothesized that East Asian children who hear and speak fraction terms in which the part-whole relation is embedded might find it easier to identify the part-whole interpretation of fractions than their non-East Asian counterparts. They tested this hypothesis in a cross-sectional study of first and second graders from Croatia, Korea, and the United States. Data were collected from different age groups of children in each nation: the first group in the middle of the first grade, the second group at the end of first grade, and the third group at the beginning of second grade. Children saw and heard each fraction name in their native language and were asked to select one from four pictorial representations that corresponded to the target fraction (see Fig. 2). All four

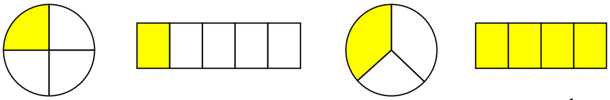


FIG. 2 Four pictorial representations from which to select one that matches $\frac{1}{4}$.

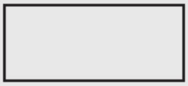
representations conveyed the part-whole meaning of fractions (e.g., a bar or a circle divided into a number of parts) with only one option matching the target fraction. The results showed that Korean children at the end of first grade already outperformed those in Croatia and the United States at the beginning of second grade. The results suggest that East Asian fraction words may facilitate children's interpretation of part-whole fractions by making a stronger association between the spoken words and corresponding part-whole visual representations. It should be noted that these results await replication (cf., Paik & Mix, 2003).

Terms for Geometric Shapes

English terms for geometric shapes are derived from Greek and Latin words. As shown in Table 3, these terms tend to be long and complicated. For young children learning shape names, these are all new words with little hint embedded in the names to link to visual representations. This is particularly true for shapes with four or more angles. In contrast, Chinese-based geometric terms are relatively straightforward. For example, pentagon is a five-angle shape (5角形) in Japanese. Aside from rectangle, all standard 2-D shapes that children see in the shape chart take a form of "(the number of angles)-angle shape."[3] That is, the meaning of each of these shapes is embedded in its term. Children learning these simple terms should have an easier time linking words to visuals and their properties than those who must memorize complex terms. To the best of my knowledge, no cross-cultural studies have examined this relation. The only cross-cultural studies in this general area of spatial terms examined young English- and Korean-speaking children's categorization of spatial words such as "in" and "on" (Choi & Bowerman, 1991; Choi, McDonough, Bowerman, & Mandler, 1999). Using a preferential looking paradigm, Choi et al. (1999) found that two groups of children as young as 18 months old showed sensitivity to language-specific spatial categories. Coupled with recent findings that spatial language supports children's encoding of spatial properties such as big, tall, and curvy and predicts their later performance on spatial reasoning (Pruden, Levine, & Huttenlocher, 2011), it is conceivable that differences in geometric shape names between English and Japanese could result in different rates of acquisition of spatial properties.

3. 3 D shape names also express property meanings more clearly in Japanese than in English (e.g., six angle pillar vs hexagonal prism).

TABLE 3 2D geometric shape names in English and Japanese

Shape	English	Japanese
	Triangle	Three angle shape 三角形
	Square	True (correct) four angle shape 正四角形
	Rectangle	Long side shape 長方形
	Pentagon	Five angle shape 五角形
	Hexagon	Six angle shape 六角形
	Octagon	Eight angle shape 八角形

Note: Square in Japanese is also expressed as true (correct) shape (正方形).

In summary, number names, fraction terms, and geometric shape names in East Asian languages facilitate children's learning of difficult mathematics concepts. Geometric shapes are given names that refer to the number of angles. Fractions are spoken in a way to convey the part-whole meaning. Finally, number names for two-digit numbers are congruent with the base-10 numeration system, which not only helps children to develop place value understanding with relative ease but also allows for the teaching of multidigit column computations to take advantage of the number naming system. Taken together, characteristics of East Asian languages make mathematics accessible to the speakers of such languages.

In contrast, children speaking non-East Asian languages are tasked with developing a link between number names and their meaning on their own or through explicit instruction, overcoming whole number bias with fractions, and learning shape names that provide no information about the properties of the shape. East Asian speakers, such as Chinese, Japanese, and Korea children, bypass many of the steps that are necessary for their non-East Asian-speaking children to take. Achievement differences are thus expected to appear as early as when children are first learning number names larger than 10 (and possibly shape names). The initial achievement gap may be small. As shown in large-scale comparison studies, however, it becomes a bigger problem as children learn more complex mathematics.

MATHEMATICS ACHIEVEMENT, CENTRAL CONCEPTUAL KNOWLEDGE, AND NUMERICAL LANGUAGE

Although explaining sources of achievement differences is important, so is considering what these differences tell us about children's understanding of number systems (e.g., whole numbers and rational numbers). Case and colleagues (e.g., Case, 1998; Case & Okamoto, 1996) distinguished mathematics achievement from foundational conceptual knowledge of number. This latter knowledge is a domain-specific conceptual structure, often referred to as a *central conceptual structure* in a specific domain (e.g., Case & Okamoto, 1996). In elementary school years, mathematics achievement in the number domain is a reflection of children's mastery of school arithmetic content. In contrast, children's conceptual knowledge of whole numbers has its root in the kind of core knowledge that develops during infancy (see Okamoto, 2010, for a review). This core knowledge has been found to develop not only in human infants but also in macaque monkeys (e.g., Hauser & Carey, 2003). Furthermore, adults who grew up in cultures without demands for enumeration skills also possess this core knowledge (e.g., Pica, Lemer, Izard, & Dehaene, 2004).

If core knowledge provides foundations for children's central conceptual knowledge, it is possible that central numerical knowledge develops at similar rates in cultures where enumeration skills are valued. However, the particular skills children are encouraged to attain could differ, which might explain large differences in achievement. In a series of studies, we demonstrated that this possibility holds up in general. Evidence counter to this general trend was found among 6-year-old children in Japan who exhibited numerical knowledge 1–2 years ahead of their US peers who, otherwise, showed age-appropriate conceptual development. Among many competing explanations, I have come to conclude that variations in numerical language characteristics play a major role in young Japanese children's acquisition of central numerical knowledge.

Development of Core Conceptual Knowledge

Infants may come into this world with the ability to detect numerical features of the world. However, it takes a few more years before children are able to count objects reliably (e.g., Gelman, 1978) or compare quantities (e.g., Starkey, 1992). As Gelman and Gallistel (1978) have shown, the act of counting requires children to acquire several important principles. For example, children must apply number names in the same order every time they count, count one item at a time without skipping any (and not counting the same item more than once), and know that the last number name spoken represents the cardinal value of the set. In other words, children must learn to adhere to cultural conventions (including number names) associated with the act of counting. Building on Gelman and Gallistel's work, Case (1992a) proposed

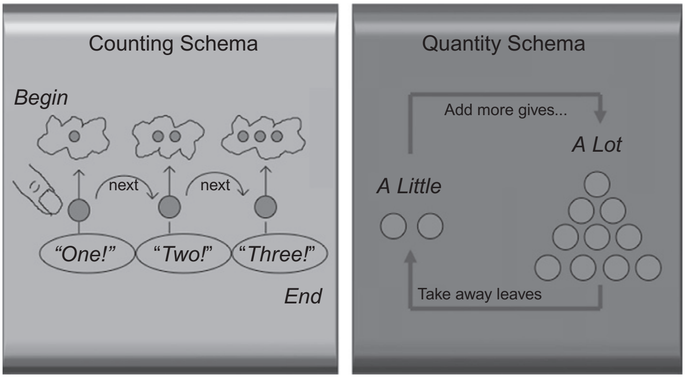


FIG. 3 Four year olds' counting schema (left) and quantity schema (right).

that by 4 years of age, children develop a schema that allows them to count a small number of items without error (Fig. 3(left)). At the same time, children develop a global quantity schema (Fig. 3(right)). This schema allows preschool-age children to compare quantities and understand the effects of quantity transformations such as addition and subtraction of small numbers (Starkey, 1992; Wynn, 1992). These two schemas provide foundations for preschoolers to interpret the quantitative world around them.

As children acquire more experience working with numbers and quantities in a broad range of situations, they learn to use counting as part of quantitative reasoning. For example, when asked which side of a balance scale (each holding different numbers of weights) would tilt, children by kindergarten or first grade realize that counting is a useful way to make quantitative judgments. In contrast, children in preschool years often fail to count the number of weights but instead visually inspect which pile of weights looks higher (Case, Okamoto, Henderson, McKeough, & Bleiker, 1996). Observations such as these led Case and colleagues to conclude that by around 6 years of age, children typically develop central numerical knowledge that results from the integration of the two earlier schemas (e.g., Case & Okamoto, 1996; see Fig. 4). This knowledge structure is often referred to as a "mental number line" (Case, 1996a).

As shown in Fig. 4, to develop a functional understanding of a mental number line requires that children understand the correspondence between Arabic numerals (written and spoken) and their magnitudes. This point was recently elaborated by Siegler, Thompson, and Schneider (2011). They reviewed studies of numerical magnitude estimation and concluded that it is not until 5 or 6 years of age that children develop a mental number line for single-digit numbers. They further articulated that the acquisition of this initial mental number line provides a foundation for children's understanding of the whole number system and the rational number system. (See also Chapter 5

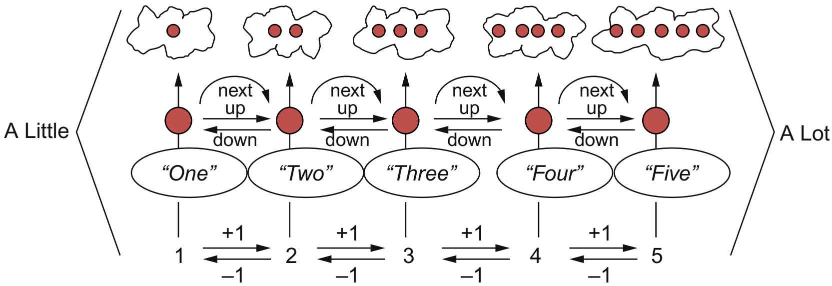


FIG. 4 Six year olds' central conceptual knowledge (mental number line).

by Opfer et al. for a theory that the cross-national gap in mathematics achievement may be attributable to a difference in the representation of symbolic numerical magnitude that is primarily influenced by early instructional inputs.)

Case and Okamoto (1996) hypothesized that the acquisition of a mental number line provides a conceptual foundation through which children interpret the numerical attributes of the world. That is, it serves as a tool to create new knowledge necessary to interpret cultural concepts of time telling, currency system, and distribution of resources, and to take advantage of instruction in school. It should be emphasized that children's experiences are culturally bound.

By about 8 years of age, children's mastery of the mental number line becomes sufficiently fluent, allowing them to focus on and tentatively begin relating two mental number lines. As a result, new properties of numerical systems such as the base-10 system become part of their understanding. Children's understanding at this level, however, is limited to interpreting the relation between ones and tens. That is, 8-year olds have a difficult time generalizing the rules of base-10 to numbers beyond 99. By about age 10, children begin to generalize the base-10 rules to three-digit numbers and possibly beyond. They are also able to develop the notion that the numerical rules and operations developed earlier can be treated as objects of manipulations to carry out complex arithmetic reasoning. For example, in comparing the numerical magnitudes of $(8-3)$ and $(9-5)$ mentally, children of this age are able to treat the result of one computation *5* $(8-3=5)$ as a new object to compare with the result of another computation *4* $(9-5=4)$. Fig. 5 shows a schematic sketch of the developmental progression.

Cultural Comparisons of Central Conceptual Knowledge and Specific Skills

Central numerical knowledge that develops during elementary school years is shaped by both biologically based factors (e.g., neural development) and children's everyday cultural experiences (e.g., mastering currency). Case (1996b)

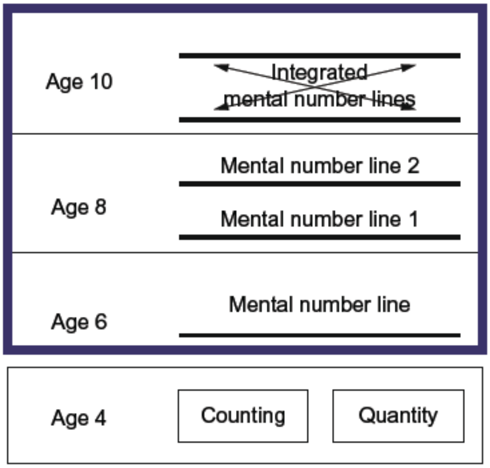


FIG. 5 A schematic representation of the development of central numerical knowledge that results from an integration of the counting and quantity schemas.

suggested that, as long as children's numerical experience includes a broad set of skills in a domain and they follow a normal course of maturation, it is conceivable that central conceptual knowledge takes a similar developmental progression across cultures. If one culture values a particular skill over others and puts more resources in helping children acquire it, children in that culture should excel in the skill emphasized. This in itself, however, does not advance central conceptual development. It is only when children have multiple opportunities to experience whole numbers in different contexts, reflect on them, and come to see numerical connections among them that these specific skills contribute to the development of central numerical knowledge.

To test this possibility, a series of cross-cultural studies were conducted. It should be noted that in Case's theory, parallel developmental progression of central conceptual knowledge takes place in different domains, such as whole numbers, space, and social/narrative knowledge (Case & Okamoto, 1996). As discussed earlier, central conceptual knowledge has its root in the core systems of knowledge on which human cognition is founded. These core systems of knowledge enable human infants to represent objects, number, space, actions, and social interactions (Spelke & Kinzler, 2007). The studies reported below focus on two such domains, space and number.

Central Spatial Knowledge and Drawing Skills

In two separate studies, children's drawings were assessed to examine their central spatial knowledge and drawing skills (Okamoto et al., 1996). In the first study, 4-, 6-, 8-, and 10-year-old Chinese and Canadian children's drawings were compared. In the second study, 6-, 8-, and 10-year-old Japanese and US children's drawings were compared. In both these studies, there were large differences in drawing skills in favor of Asian children. However, there were no statistically significant differences in how children spatially organized objects to be drawn. That is, children in all four nations developed age-appropriate central spatial knowledge. Differences only appeared in children's specific skills of drawing. In both China and Japan, drawing is viewed as an important cultural activity and included in weekly school schedules. In Canada and the United States, drawing is overshadowed by the "three R's" and in some cases completely omitted from the school schedule. In sum, cultural emphasis on drawing was seen in children's drawing skills but not in central spatial knowledge.

Central Numerical Knowledge and Mathematics Achievement

Two studies are reported that examined the relation between central numerical knowledge and mathematics achievement. The first study compared 6-, 8-, and 10-year-old children in Japan and the United States (Okamoto et al., 1996). To test Case's theory of whole number development, children were individually tested on the number knowledge and balance scale tests. The number knowledge test included items that assessed elements described in Figs. 3–5. One important feature was to develop questions that children typically do not encounter in school. All three age groups took this test. As for the balance scale test, the 6- and 10-year olds in each nation took this test. They were asked which side of the scale would tilt and why, as the number of weights and distance from the fulcrum were manipulated. The items for the balance scale test were once again designed to assess their level of central numerical knowledge. An achievement test was group administered to the 10-year-old children in each nation. The items were drawn from the fifth-grade test developed by Stevenson and colleagues (e.g., Stevenson et al., 1986).

As expected, there was a large and statistically significant achievement difference in favor of Japanese 10-year-old children. Despite this difference, no significant differences were found for the 6- or 10-year-old children in their performance on the balance scale test. As for the number knowledge test, no differences were found for the 8- and 10-year-old children. However, Japanese 6-year-old children outperformed their US counterparts. For the most part, these results show the same trend as in the drawing studies. That is, mathematics achievement (specific knowledge) valued by Japanese culture (as described in Stevenson et al., 1986, and Stigler & Hiebert, 1999) showed a substantial difference. In contrast, all age groups of children in both nations with the exception of the Japanese 6-year olds on the number knowledge test showed age-appropriate central numerical knowledge.

This cross-cultural study recruited students from middle-income families in each nation. It could be that if one tested US children who are from high-income families who attended a school well-known for its rigorous curriculum and high achievement in mathematics, the results could be different.

TABLE 4 Means (standard deviations) for the number knowledge test by national group, achievement level, and age

The United States Achievement						Japan Achievement		
Middle			*High*			*Middle*		
Age	N	*Mean*	*Age*	N	*Mean*	*Age*	N	*Mean*
6 4	22	1.82 (0.31)	6 0	25	1.69 (0.43)	6 5	21	2.52 (0.73)
8 1	23	3.04 (0.31)	8 1	26	3.12 (0.53)	8 5	26	3.33 (0.77)
10 5	24	4.23 (0.68)	10 2	20	3.79 (0.55)	10 5	20	4.05 (0.76)

Note: Hypothesized age-appropriate mean scores are 2, 3, and 4 for 6-, 8-, and 10-year-old groups, respectively.

Okamoto, Curtis, Chen, Kim, and Karayan (1997) tested this possibility. Children in Okamoto et al.'s study were given the same number knowledge test as above. As shown in Table 4, high-achieving US children were no different from their US and Japanese counterparts from middle-income families in their central numerical knowledge with the exception of the Japanese 6-year olds.[4]

Okamoto et al. (1996) suggested that central numerical knowledge is necessary for the mastery of a range of numerical skills and concepts, including those emphasized in mathematics instruction. In order to contribute to the development of central numerical knowledge, however, a broad range of experiences in the numerical domain is necessary, not just mastery of school arithmetic. Just as drawing was considered to be a specific skill in the spatial domain, mathematics achievement is a specific skill in the numerical domain. Specific skills reflect children's experiences in their cultural and socioeconomic contexts.

Taken together, findings from these cross-national studies suggest that children growing up in nations as different as the United States and Japan develop age-appropriate central conceptual knowledge in spatial and whole number domains. The only exception to these general findings was the performance by Japanese 6-year olds on the number knowledge test.

4. Although children from high and middle income families did not differ in their central con ceptual knowledge, children from low income families who seldom experience activities with numerical content tend to be behind their peers in central conceptual knowledge (e.g., Griffin and Case, 1996) and estimation of numerical magnitudes (e.g., Ramani & Siegler, 2008). These researchers also showed that theory based training significantly improved children's numerical knowledge.

Effects of Numerical Language

The anomaly found in the cross-cultural studies was significantly stronger performance by Japanese 6-year olds than their US counterparts on the number knowledge test. US 6-year olds, however, showed age-appropriate performance on the number knowledge and balance scale tests. Japanese counterparts also showed age-appropriate performance on the balance scale test. Thus, it was only on the number knowledge test that Japanese 6-year olds excelled. In fact, several of these Japanese 6-year olds successfully answered the 8-year-old level items designed to require understanding of two mental numbers or the meaning of tens and ones (see Fig. 5). What do these results mean? Among many competing explanations, I favor the numerical language explanation. As the data from Miura and colleagues' studies showed, East Asian speakers, including Japanese children, develop an understanding of the meaning of tens and ones earlier than their non-East Asian peers. The number knowledge test at the 8-year-old level included items such as "which is bigger, 69 or 71?" Japanese 6-year-old children who hear "six-tens nine" and "seven-tens one" could easily distinguish tens and ones and select the correct answer. In contrast, English-speaking 6-year olds do not have this language advantage. They often mistakenly attend to the number words in the one's column to respond to this sort of question. In contrast, the variables in the balance scale (i.e., the number and location of weights) used numbers less than 10. Thus, there was no advantage for Japanese 6-year olds on this test.

As number words are used to solve many numerical situations in everyday life, it is conceivable that quality and quantity of experience to develop base-10 knowledge helped advance Japanese 6-year olds' central numerical knowledge. The data presented in Case et al. (1996) provide a clue as to if this is a possibility. To verify the existence of the central numerical knowledge, Case et al. developed a battery of six tests in the numerical domain. The 8-year-old-level items on the three of the tests (including the number knowledge test) used two-digit numbers, whereas the three other tests (including the balance scale test) used only single-digit numbers. When this battery was administered to 6-year-old children in the United States, Case et al. found a single latent factor underlying children's numerical thought. It is conceivable that two latent factors might result if this battery was administered to Japanese 6-year olds. Only by administering the entire battery cross-culturally, would we know whether numerical language differentially influences the development of core numerical knowledge. It might confirm that Japanese 6-year olds' performance on the balance scale test *was* the anomaly in the data collected. Given the pattern of performance reflected in the available data, my tentative conclusion is that variations in numerical language characteristics played a major role in young Japanese children's development of central numerical knowledge.

CONCLUSIONS

In this chapter, I examined the role of mathematics language in explaining children's mathematics achievement and the development of central numerical knowledge. A particular focus was placed on children who are just beginning formal schooling. I presented evidence that Chinese, Japanese, and Korean children of this age have an easier time mastering place value because place value is embedded in their spoken language. In contrast, children who speak non-East Asian languages such as English, French, and Swedish do not have this sort of language support. Thus, they must make an explicit link among Arabic numerals, spoken words, and place value. Because the notion of place value is important in carrying out multidigit arithmetic, it follows that the early advantage of these East Asian speakers may facilitate later learning of mathematics. Of course, international comparisons of mathematics achievement go beyond just testing whole number understanding. I presented preliminary evidence that differences in mathematical language characteristics might contribute to achievement differences in other areas such as fractions and geometry.

My primary argument is that language characteristics influence specific aspects of mathematics performance. I am not proposing that these East Asian languages support all aspects of mathematics learning. In fact, when children are first learning single-digit number names, Japanese 2- to 3-year olds were found to be behind their English- and Russian-speaking peers in mastering the cardinal meaning of one, two, and three (Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007). Another study also confirmed that Japanese 2-year olds were behind their English-speaking peers in small number comprehension (Barner, Libenson, Cheung, & Takasaki, 2009). Japanese toddlers' difficulty understanding single-digit numbers is due, in part, to the dual counting systems of Japanese number words 1–10. There are two sets of counting words for these numbers in Japanese (see Okamoto, 2015, for a review). It is understandable if Japanese toddlers are delayed because in everyday life they hear two different number words, say "ichi" and "hi" for 1.[5] These initial difficulties, however, are quickly overcome by 4-year olds, and Japanese preschoolers master counting principles just like their English-speaking peers do.

About the time children begin formal schooling, important conceptual development takes place (Case, 1996a). As described in this chapter, by about 6 years of age, children across cultures with demands for enumeration skills develop central numerical knowledge that allows them to mentally reason about quantitative relations with numbers. Although 6-year olds' mental

5. Although number words in Table 1 are more commonly used in everyday life and in learning mathematics, there exist indigenous Japanese counting words for 1 10. Japanese toddlers are exposed to these two different ways of counting from early on (e.g., "ichi mai" for one sheet of paper and "hi tori" for one person).

arithmetic is typically limited to single-digit numbers, the data from cross-cultural studies pointed to the possibility that Japanese 6-year olds were beginning to show mental manipulations of two-digit numbers. The cross-cultural studies provided evidence that despite large achievement differences, central conceptual knowledge develops at similar rates across cultures with the exception of the just-mentioned Japanese 6-year olds.

As pointed out in previous studies, children in Japan spend more time studying mathematics than their US peers (e.g., Stevenson et al., 1986), and mathematics teaching in Japan is qualitatively different from teaching in the United States (Stigler & Hiebert, 1999). Furthermore, children's experience with numbers in and out of school is different, due in part to numerical language characteristics. Take, for example, children's acquisition of the currency system. Children in all cultures with trade learn to use their currency to determine how much to pay and how much change to receive. However, the currency systems themselves differ. The coins in the United States, for example, include a penny, a dime, and a quarter, and a 20-dollar bill is commonly used, whereas in Japan, they are one yen, five yen, ten yen, five-tens yen, one-hundred yen, and five-hundreds yen, and there is no bill equivalence to the US $20 bill. The Japanese currency system thus reinforces children's acquisition of tens and ones in everyday experience of shopping.

Cooking is also an activity practiced in all cultures. Japanese children learn to use the metric system using base-10 spoken number words. This is not the case in the United States. These are just some of the everyday experiences in which children speak regular or irregular number words. Japanese 6-year olds are developing specific numerical skills (e.g., money knowledge and metric system), all of which use the regular base-10 number words. It is thus reasonable to expect that Japanese 6-year olds' acquisition of specific skills contributes to the development of central numerical knowledge. Until further data show otherwise, I conclude that spoken Japanese number words that include clear markers for place value influence not only mathematics achievement but also the development of central numerical knowledge for this age group.

REFERENCES

Alsawaie, O. N. (2004). Language influence on children's cognitive representation of number. *School Science and Mathematics*, *104*, 105 111.

Barner, D., Libenson, A., Cheung, P., & Takasaki, M. (2009). Cross linguistics relations between quantifiers and numerals in language acquisition: Evidence from Japanese. *Journal of Exper imental Child Psychology*, *103*, 421 440.

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1997). *Mathematics achievement in the middle School Years: IEA's third international mathematics and science study (TIMSS).* Chestnut Hill, MA: TIMSS International Study Center, Boston College.

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296 333). New York, NY: Macmillan.

Brown, J. S., & Burton, R. R. (1978). Diagnostic models for procedural bugs in basic mathemati cal skills. *Cognitive Science*, *2*, 155 192.

Case, R. (1992). *The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Case, R. (1996a). Introduction: Reconceptualizing the nature of children's conceptual structures and their development in middle childhood. In R. Case & Y. Okamoto (Eds.), *Monographs of the Society for research in child development: Vol. 61 (1 2). The role of central conceptual structures in the development of children's thought* (pp. 1 26) [Serial No. 246].

Case, R. (1996b). Modeling the dynamic interplay between general and specific change in children's conceptual understanding. In R. Case & Y. Okamoto (Eds.), *Monographs of the Society for research in child development: Vol. 61 (1 2). The role of central conceptual structures in the development of children's thought* (pp. 156 188) [Serial No. 246].

Case, R. (1998). The role of central conceptual structures in the development of children's scien tific and mathematical thought. In A. Demetriou, R. Shayer, & A. Elfklides (Eds.), *Neo Piagetian theories of cognitive development: Implications and applications for education* (pp. 52 64). London: Routledge.

Case, R., & Okamoto, Y. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for research in child development: Vol. 61 (1 2)*. [Serial No. 246].

Case, R., Okamoto, Y., Henderson, B., McKeough, A., & Bleiker, C. (1996). Exploring the mac rostructure of children's central conceptual structures for number and narrative. In R. Case & Y. Okamoto (Eds.), *Monographs of the Society for research in child development: Vol. 61 (1 2). The role of central conceptual structures in the development of children's thought* (pp. 59 82) [Serial No. 246].

Choi, S., & Bowerman, M. (1991). Learning to express motion events in English and Korea: The influence of language specific lexication patterns. *Cognition*, *41*, 83 121.

Choi, S., McDonough, L., Bowerman, M., & Mandler, J. (1999). Early sensitivity to language specific spatial categories in English and Korean. *Cognitive Development*, *14*, 241 268. https://doi.org/10.1016/s0885 2014(99)00004 0.

Dowker, A., Bala, S., & Lloyd, D. (2008). Linguistics influence of mathematical development: How important is the transparency of the counting system? *Philosophical Psychology*, *21*, 523 538.

Dowker, A., & Roberts, M. (2015). Does the transparency of the counting system affect children's numerical abilities? *Frontiers in Psychology*, *6*, 945. https://doi.org/10.3389/fpsyg.2015.00945.

Gelman, R. (1978). Counting in the preschoolers: What does and does not develop. In R. Siegler (Ed.), *Children's thinking: What develops?* New Jersey: Lawrence Erlbaum Associates.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Hauser, M. D., & Carey, S. (2003). Spontaneous representations of small numbers of objects by rhesus macaques: Examinations of content and format. *Cognitive Psychology*, *47*, 367 401.

Helmreich, I., Zuber, J., Pixner, S., Kaufmann, L., Nuerk, H. C., & Moeller, K. (2011). Language effects on children's mental number line: How cross cultural differences in number word sys tems affect spatial mappings of numbers in a non verbal task. *Journal of Cross Cultural Psychology*, *42*, 598 613.

Ho, C. S., & Cheng, F. S. (1997). Training in place value concepts improves children's addition skills. *Contemporary Educational psychology*, *22*, 495 506.

Ho, C. S., & Fuson, K. C. (1998). Children's knowledge of teen quantities as tens and ones: Com parisons of Chinese, British, and American kindergartners. *Journal of Educational Psychol ogy*, *90*, 536 544.

Husén, T. (Ed.), (1967). *International study of achievement in mathematics: A comparison of twelve countries*. Stockholm: Almqvist & Wiksell. Vols. 1 2.

Kieren, T. E. (1993). Rational and fractional number: From quotient fields to recursive under standing. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 261 288). Hillsdale, NJ: Erlbaum.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of funda mental mathematics in China and the United State*. Mahwah, NJ: Lawrence Erlbaum Associates.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for Research in Mathematics Education*, *21*, 16 32.

Miura, I. T. (1987). Mathematics achievement as a function of language. *Journal of Educational Psychology*, *79*, 79 82.

Miura, I. T., Kim, C. C., Chang, C. M., & Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representation of number: Cross national comparisons. *Child Devel opment*, *59*, 1445 1450.

Miura, I. T., & Okamoto, Y. (1989). Comparisons of U.S. and Japanese first graders' cognitive representation of number and understanding of place value. *Journal of Educational Psychol ogy*, *81*, 109 113.

Miura, I. T., & Okamoto, Y. (1999). Counting in Chinese, Japanese, and Korean: Support for number understanding. In C. A. Edwards (Ed.), *Perspectives on Asian Americans and Pacific Islanders* (pp. 29 36) of *Changing the Faces of Mathematics*. In W. G. Secada, (Series Ed.). Reston, VA: National Council of Teachers of Mathematics.

Miura, I. T., & Okamoto, Y. (2003). Language supports for mathematics understanding and per formance. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: The construction of adaptive expertise* (pp. 229 242). Mahwah, NJ: Lawrence Erlbaum Associates.

Miura, I. T., Okamoto, Y., Kim, C. C., Steere, M., & Fayol, M. (1993). First graders' cognitive repre sentation of number and understanding of place value: Cross national comparisons France, Japan, Korea, Sweden, and the United States. *Journal of Educational Psychology*, *85*(1), 24 30.

Miura, I. T., Okamoto, Y., Vlahovic Stetic, V., Kim, C. C., & Han, J. H. (1999). Language sup ports for children's intuitive understanding of numerical fractions: Cross national compari sons. *Journal of Experimental Child Psychology*, *74*, 356 365.

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H. C. (2011). Early place value under standing as a precursor for later arithmetic performance A longitudinal study on numerical development. *Research in Developmental Disabilities*, *32*(5), 1837 1851.

Moseley, B., & Okamoto, Y. (2008). Identifying fourth graders' linking of rational number repre sentation: A mixed method approach. *School Science and Mathematics Journal*, *108*, 238 250.

Moseley, B., Okamoto, Y., & Ishida, J. (2007). Comparing U.S. and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science and Mathematics Education*, *5*(1), 165 185.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, *30*, 122 147.

Mullis, I. V. S., Martin, M. O., & Foy, P. (with Olson, J. F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.). (2008). TIMSS 2007 international mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College. Retrieved from http://timss.bc.edu/TIMSS2007/mathreport.html.

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1998). *Mathematics achievement in the primary school years: IEA's Third International Mathematics and Science Study (TIMSS)*. Chestnut Hill, MA: TIMSS International Study Center, Boston College.

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College Retrieved from: http://timss.bc.edu/timss2011/international results mathematics.html.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2005). *TIMSS 2003 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades*. Chestnut Hill, MA: TIMSS and PIRLS International Study Center, Boston College Retrieved from: http://timss.bc.edu/timss2003i/scienceD.html.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., … Smith, T. A. (2000). *TIMSS 1999 international mathematics report: Findings from IEA's repeat of the third international mathematics and science study at the eighth grade*. Chestnut Hill, MA: TIMSS and PIRLS International Study Center, Boston College. Retrieved from: http://timss.bc.edu/timmathbench report/mathbench report/t99b math report.html.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*, 27 52.

Okamoto, Y. (2010). Children's developing understanding of number: Mind, brain, and culture. In M. Ferrari & L. Vuletic (Eds.), *Developmental relations among mind, brain, and education: Essays in honor of Robbie Case* (pp. 129 148). New York, NY: Springer.

Okamoto, Y. (2015). Mathematics learning in the USA and East Asia: Influences of language. In R. Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 415 429). Oxford: Oxford University Press.

Okamoto, Y., Case, R., Bleiker, C., & Henderson, B. (1996). Cross cultural investigations. In R. Case & Y. Okamoto (Eds.), *Monographs of the Society for research in child development: Vol. 61 (1 2). The role of central conceptual structures in the development of children's thought* (pp. 131 155) [Serial No. 246].

Okamoto, Y., Curtis, R., Chen, P. C., Kim, S., & Karayan, S. (1997). *A cross national comparison of children's intuitive number sense: its development and relation to formal mathematics learning: Final report* [submitted to Spencer Foundation].

Paik, J. H., & Mix, K. S. (2003). U.S. and Korean children's comprehension of fraction names: A reexamination of cross national differences. *Child Development*, *74*, 144 154.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*, 499 503.

Princiotta, D., Flanagan, K. D., & Germino Hausken, E. (2006). *Fifth grade: Findings from the fifth grade follow up of the early childhood longitudinal study, kindergarten class of 1998 99 (ECLS K)*. [NCES 2006 038]. Washington, DC: National Center for Education Statistics, U.S. Department of Education.

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? *Developmental Science*, *14*, 1417 1430. https://doi.org/10.1111/j.1467 7687.2011.01088.x.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low income children's numerical knowledge through playing number board games. *Child Development*, *79*, 375 394. https://doi.org/10.1111/j.1467 8624.2007.01131.x.

Sarnecka, B. W., Kamenskaya, V. G., Yamana, Y., Ogura, T., & Yudovina, Y. B. (2007). From grammatical numbers to exact numbers: Early meanings of 'one', 'two', and 'three' in English, Russian, and Japanese. *Cognitive Psychology*, *55*, 136 168.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young students. *Child Development*, *75*(2), 428 444.

Siegler, R. S., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., … Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide (NCEE #2010 4039)*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from: http://ies.ed.gov/ncee/wwc/PracticeGuide/15.

Siegler, R. S., Duncan, G. J., David Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*, 691 697.

Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics. *Psychological Science*, *19*, 759 763.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273 296.

Spelke, E. S., & Kinzler, K. D. (2007). Core knowledge. *Developmental Science*, *10*, 89 96.

Starkey, P. (1992). The early development of numerical reasoning. *Cognition*, *43*, 93 126.

Stevenson, H. W., Lee, S. Y., & Stigler, J. W. (1986). Mathematics achievement of Chinese, Japanese, and American children. *Science*, *231*, 693 699.

Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS Videotape Classroom study: Methods and findings from an exploratory research project on eighth grade mathematics instruction in Germany, Japan, and the United States*. Washington, DC: U.S. Department of Education.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: The Free Press.

Stigler, J. W., Lee, S. Y., & Stevenson, H. W. (1987). Mathematics classrooms in Japan, Taiwan, and the United States. *Child Development*, *58*, 1272 1285.

Towse, J. N., Muldoon, K., & Simms, V. (2015). Figuring out children's number representations: Lessons from cross cultural work. In A. Dowker & R. C. Kadosh (Eds.), *Oxford Handbook of Numerical Cognition* (pp. 402 414). New York, NY: Oxford University Press.

Towse, J., & Saxton, M. (1997). Linguistic influences on children's number concepts: Methodological and theoretical considerations. *Journal of Experimental Child Psychology*, *66*, 362 375.

Vosniadou, S., Vamvakoussi, X., & Skopeliti, I. (2008). The framework theory approach to conceptual change. In S. Vosniadou (Ed.), *International handbook of research on conceptual change* (pp. 3 34). Mahwah, NJ: Erlbaum.

Wynn, K. (1992). Children's acquisition of number words and the counting system. *Cognitive Psychology*, *24*, 220 251.